INTEGRATION BEE FINALS

The finals are conducted in rounds. One at a time, each remaining contestant will have two and a half minutes to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

Contestants must circle their final answer. Contestants do not need to include the constant of integration $+C$ in their answer.

The last person remaining wins an additional $75 and will be crowned the Integration Champion!

INTEGRAL #1

READY,

GET SET,...

2:30

INTEGRAL #1

$$\int \frac{x^2}{1} \frac{x^6}{x^2} dx$$

INTEGRAL #1

$$\int \frac{x^2 \quad x^6}{1 \quad x^2}\,dx$$

$$= \int \frac{x^2(1 \quad x^2)(1+x^2)}{1 \quad x^2}\,dx$$

$$= \int (x^2 + x^4)\,dx$$

$$= \boxed{\frac{x^3}{3} + \frac{x^5}{5} + C}$$

INTEGRAL #2

READY,

GET SET,...

2:30

INTEGRAL #2

$$\int \frac{1 + \frac{x}{x+1}}{}$$

INTEGRAL #3

READY,

GET SET,...

2:30

INTEGRAL #3

$$\int \frac{x}{x^4 + 2x^2 + 1} dx$$

INTEGRAL #3

$$\int \frac{x}{x^4+2x^2+1}\,dx$$

$$= \int \frac{x}{(x^2+1)^2}\,dx$$

$$= \frac{1}{2}\int \frac{1}{u^2}\,du \qquad u = x^2+1, \quad du = 2x\,dx$$

$$= \frac{1}{2u} + C$$

$$= \boxed{\frac{1}{2(x^2+1)} + C}$$

READY,

GET SET,...

2:30

INTEGRAL #4

$$\int \frac{x^2 + x \quad 1}{(2x+1)^2 \quad 5}\,dx$$

$$= \int \frac{x^2 + x \quad 1}{4x^2 + 4x \quad 4}\,dx$$

$$= \int \frac{1}{4}\,dx$$

$$= \boxed{\frac{1}{4}}$$

INTEGRAL #5

READY,

GET SET,...

2:30

INTEGRAL #5

$$\int \frac{1}{\sqrt{x+7+\sqrt{x}}}\,dx$$

INTEGRAL #5

$$\int \frac{1}{\sqrt{x+7}+\sqrt{x}}\,dx$$

$$= \int \frac{1}{\sqrt{x+7}+\sqrt{x}} \cdot \frac{\sqrt{x+7}-\sqrt{x}}{\sqrt{x+7}-\sqrt{x}}\,dx$$

$$= \int \frac{\sqrt{x+7}-\sqrt{x}}{7}\,dx$$

$$= \boxed{\frac{2}{21}(x+7)^{3=2} - x^{3=2} + C}$$

INTEGRAL #6

READY,

GET SET,...

2:30

INTEGRAL #6

$$\int \sqrt{\frac{1}{4 + x^2}} \cdot \left(\frac{1}{x^2}\right)^2 \, dx$$

INTEGRAL #6

$$\int \sqrt{4 + \left(x^2 - \frac{1}{x^2}\right)^2}\, dx$$

$$= \int \sqrt{4 + x^4 \left(2 + \frac{1}{x^4}\right)}\, dx$$

$$= \int \sqrt{\left(x^2 + \frac{1}{x^2}\right)^2}\, dx$$

$$= \int \left(x^2 + \frac{1}{x^2}\right) dx = \boxed{\frac{x^3}{3} - \frac{1}{x} + C}$$

READY,

GET SET,...

2:30

INTEGRAL #7

$$\int \frac{e^{(2x\ \ 1)=x}}{x^2}\,dx$$

$$= \int \frac{e^{2\ \ 1=x}}{x^2}\,dx$$

$$= \int e^u\,du \qquad u = 2\ \ 1=x,\ du = \frac{1}{x^2}\,dx$$

$$= e^u + C$$

$$= \boxed{e^{2\ \ 1=x} + C}$$

READY,

GET SET,...

2:30

INTEGRAL #8

$$\int \frac{e^x}{e^x + 1} dx$$

READY,

GET SET,...

2:30

$$\int \tan x \ln \cos x \, dx$$

INTEGRAL #9

$$\int \tan x \ln\cos x \, dx$$

$$= \int u \, du \qquad u = \ln\cos x, \quad du = \frac{1}{\cos x}(-\sin x)\,dx = \tan x \, dx$$

$$= \frac{u^2}{2} + C$$

$$= \boxed{\frac{1}{2}(\ln\cos x)^2 + C}$$

READY,

GET SET,...

2:30

$$\int \tan x \sec^4 x \sqrt{1 + \sec^4 x}\, dx$$

INTEGRAL #10

$$\int \tan x \sec^4 x \sqrt{1+\sec^4 x}\, dx$$

$$= \frac{1}{4}\int \sqrt{u}\, du \qquad u = 1+\sec^4 x, \quad du = 4\sec^3 x(\sec x \tan x)\, dx$$

$$= \frac{1}{6}u^{3=2} + C$$

$$= \boxed{\frac{1}{6}\left(1+\sec^4 x\right)^{3=2} + C}$$

READY,

GET SET,...

2:30

INTEGRAL #11

$$\int \cos^5 x \sin 2x \, dx$$

INTEGRAL #11

$$\int \cos^5 x \sin 2x \, dx$$

$$= \int \cos^5 x (2 \sin x \cos x) \, dx$$

$$= 2 \int \cos^6 x \sin x \, dx \qquad u = \cos x, \quad du = \sin x \, dx$$

$$= 2 \int u^6 \, du$$

$$= \frac{2}{7} u^7 + C = \boxed{\frac{2}{7} \cos^7 x + C}$$

INTEGRAL #12

$$\int \frac{2x^2 + 7x + 4}{x + 3} \, dx$$

INTEGRAL #13

READY,

GET SET,...

2:30

READY,

GET SET,...

2:30

INTEGRAL #14

$$\int x \csc^2 x \, dx$$

$$\int x \csc^2 x \, dx$$

$$= x \cot x + \int \cot x \, dx$$

Integration by parts:

$u = x \qquad dv = \csc^2 x \, du$

$du = dx \qquad v = \cot x$

READY,

GET SET,...

2:30

INTEGRAL #15

$$\int \frac{x}{x^3 + 3x^2 + 3x + 1} \, dx$$

INTEGRAL #15

$$\int \frac{x}{x^3+3x^2+3x+1}\,dx$$

$$= \int \frac{x}{}$$